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The Equivalence of Second Order Impedance Control and Proportional Gain Explicit Force Control: Theory and Experiments

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Abstract

This paper reveals the essential equivalence of second order impedance control and proportional gain explicit force control with feedforward. This is first done analytically by reviewing each control method and showing how they mathematically correspond. For stiff environments the correspondence is exact. However, even for softer environments similar response of the system is indicated. Next, the results of an implementation of these control schemes on the CMU DD Arm II are presented, confirming the predictions of the analysis. These results experimentally demonstrate that proportional gain force control and impedance control, with and without dynamics compensation, have equivalent response to commanded force trajectories.

1 Introduction

There is a whole class of tasks that seem to implicitly require controlling the force of interaction between a manipulator and its environment: pushing, scraping, grinding, pounding, polishing, twisting, etc. Thus, force control of the manipulator becomes necessary in at least one of the degrees of freedom of the manipulator; the other degrees of freedom remain position controlled. Mason formalized this idea and called it Hybrid Control [14]. Simply put, the manipulator should be force controlled in directions in which the position is constrained by environmental interaction, and position controlled in all orthogonal directions.

The Hybrid Control formalism does not specify what particular type of position or force control should be used. It only partitions the space spanned by the total degrees of

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Figure 1: Explicit force control block diagram.

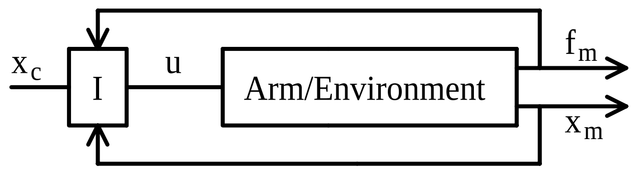


Figure 2: Impedance control block diagram.

freedom into one subspace in which position control is employed, and another in which force control is employed. In the position control subspace simple strategies have proven adequate (eg. PID), while sophisticated enhancements have improved performance (eg. computed torque control, adaptive control). However, in the force control subspace, two main conceptual choices have emerged: *explicit force control* and *impedance control*. Figures 1 and 2 are simple block diagrams of these two types of control schemes. The major difference between these schemes is the commanded value: explicit force control requires commanded force, while impedance control requires commanded position. In order for these to be feedback controllers, explicit force control needs force measurement, while impedance control needs position measurement. In addition, impedance control requires force measurement — without it an impedance controller reduces to a position controller.

Ideally, an explicit force controller attempts to make the manipulator act as pure force source, independent of position. Like position control, the obvious first choice has been some manifestation of PID control (i.e. P, PD, PI, etc.). These have met with varying amounts of success depending on the characteristics of the particular actuators, arm links, sensor, and environment.

Alternatively, impedance control has been presented as a method of stably interacting with the environment. This is achieved by providing a *dynamic* relationship between the robot's position and the force it exerts. A complete introduction to impedance control is beyond the scope of this discussion and the reader is referred to the previous work of other researchers [6, 8]. The basic tenet of impedance control is that the arm should be controlled so that it behaves as a mechanical impedance to positional constraints imposed by the environment. This means that the force commanded to the actuators is dependent on its position: $f = \mathcal{Z}(x)$, where $\mathcal{Z}$ may be a function or an operator. If the impedance is linear, it can be represented in the Laplace domain as $F(s) = Z(s)X(s)$; the order of Z determines the order of the impedance controller. The resultant behavior

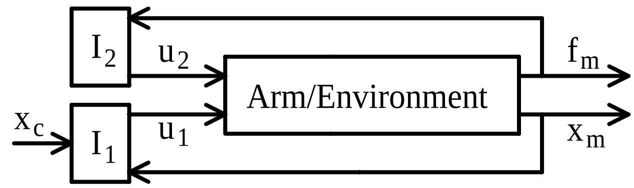


Figure 3: Impedance control block diagram with the controller divided into its position part, I_1, and its force part, I_2.

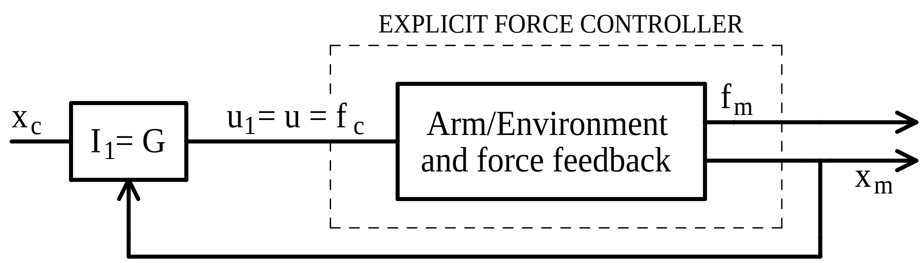


Figure 4: Impedance control block diagram redrawn to show the inner explicit force controller.

of the manipulator is obvious: if it is unconstrained it will accelerate; if it is constrained the forces from the actuators will be transmitted through the arm and exerted on the environment. In this paper we will only consider the second case of physical interaction in which force feedback must be used in the control of the mechanical impedance of the manipulator. For linear impedance relationships, the force feedback loop may be separated as in Figure 3. It can be seen that this figure may be further modified as in Figure 4 to show that the force feedback loop is part of an internal explicit force controller. Thus, an impedance controller that utilizes force feedback contains an explicit force controller. Further, when the arm is against a stiff environment, the feedback term, x_m, is constant and may be ignored. Thus, impedance control reduces to explicit force control.

This paper explores the exact correspondence between explicit force control and impedance control. In particular, it is shown that impedance controllers that utilize force feedback must be second order; lesser order impedance relations are essentially open-loop to force. Analysis of the second order impedance controller reveals that it has an algebraic structure akin to proportional gain explicit force control with feedforward. This correspondence becomes exact when the position feedback is constant. In practice, this criterion is regularly met by stiff environments, or soft environments in equilibrium with the arm.

We have implemented both second order impedance control with and with out dynamics compensation, as well as proportional gain explicit force control with feedforward. These implementations were in six DOF on the CMU DDarm II. The results reveal the

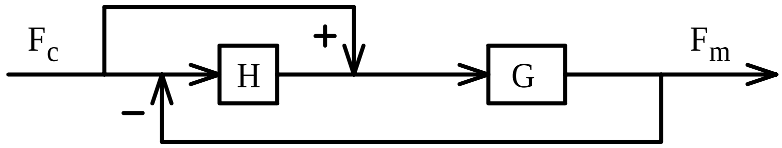


Figure 5: Proportional gain explicit force control block diagram.

equivalent response of the impedance and explicit force control strategies, even for the case of soft environment contact. These results experimentally confirm what is analytically indicated: the equivalence of second order impedance control and proportional gain force control.

This paper is organized as follows. First, proportional gain explicit force control is reviewed and analyzed. It is also shown how the proportional gain values can be as low as negative one. Second, impedance control, with and without dynamics compensation, is reviewed and analyzed. It is shown that only second order impedance control utilizes force feedback information. It is also shown that second order impedance control employs proportional gain explicit force control, and that for stiff environments the two become the same controller. In the last part of this paper, the insights and predictions from this analysis of the controllers is experimentally verified.

2 Proportional Gain Explicit Force Control

The first controller to be discussed is proportional gain explicit force control. The chosen form of this controller is:

$$f \;=\; f_c \;+\; K_{fp}(f_c - f_m) \;-\; K_v \dot{x}_m \tag{1}$$

where subscripts c and m denote the commanded and measured quantities, respectively. The feedforward term, f_c, is necessary to provide a bias force when the force error is zero — without it the system is guaranteed to have a steady state force error. The velocity gain, K_v, adds damping to the system. A block diagram for this controller is shown in Figure 5, where $H = K_{fp}$ and G is the arm / sensor / environment plant and includes the active damping control loop. It has previously been shown that G can be considered a fourth order transfer function [3], and we have experimentally extracted parameter values for the model [16, 18]. The closed loop transfer function with the feedforward term is:

$$\frac{F_c}{F_m} = \frac{(1 + K_{fp})G}{1 + K_{fp}G}. \tag{2}$$

This is a Type 0 System and will have a nonzero steady-state error for a step input. The root locus of this system is shown in Figures 6 and 7. The corresponding Bode plots are shown in Figure 8. As can be seen from the root locus, proportional control makes the system more oscillatory and can make it unstable. This instability is contrary to other researchers' predictions which were the result of using a plant model that was not experimentally derived [3, 18]. The Bode plots further illustrate this problem. There is

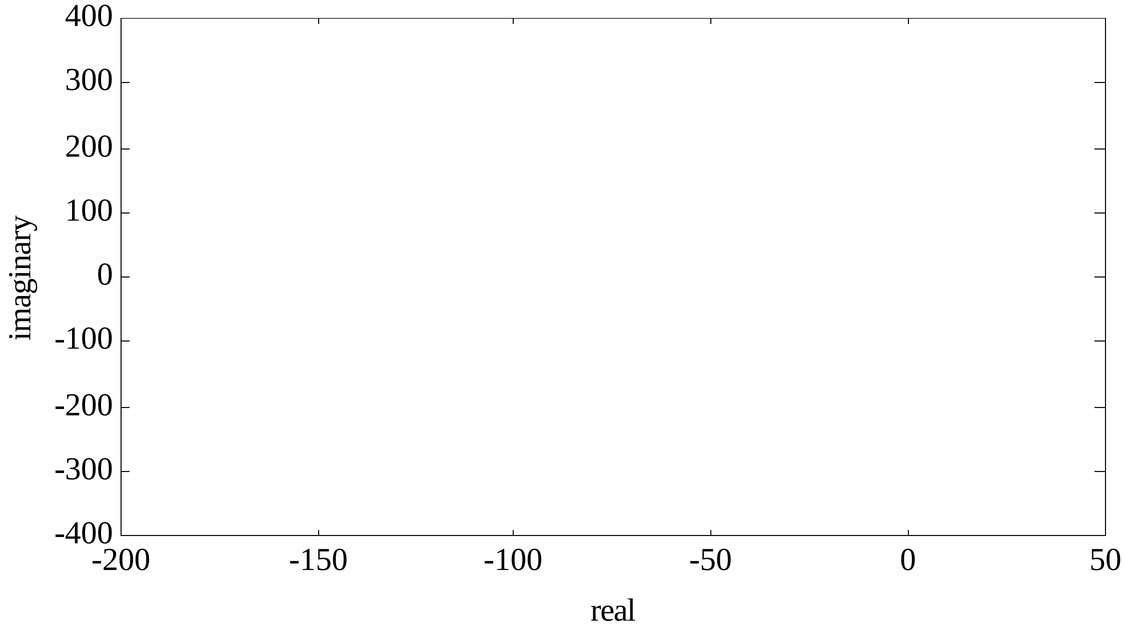


Figure 6: Root locus for the fourth order model under proportional gain explicit force control.

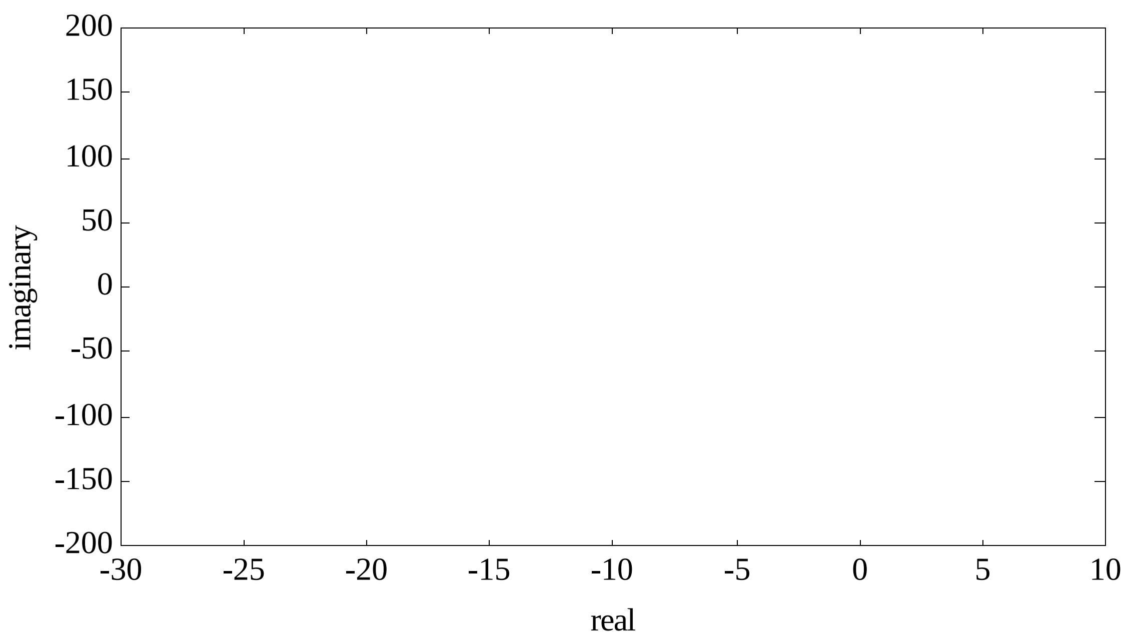


Figure 7: Enlargement of root locus in Figure 6 with K_{fp} values of 0 to 1.5 in steps of 0.1 .

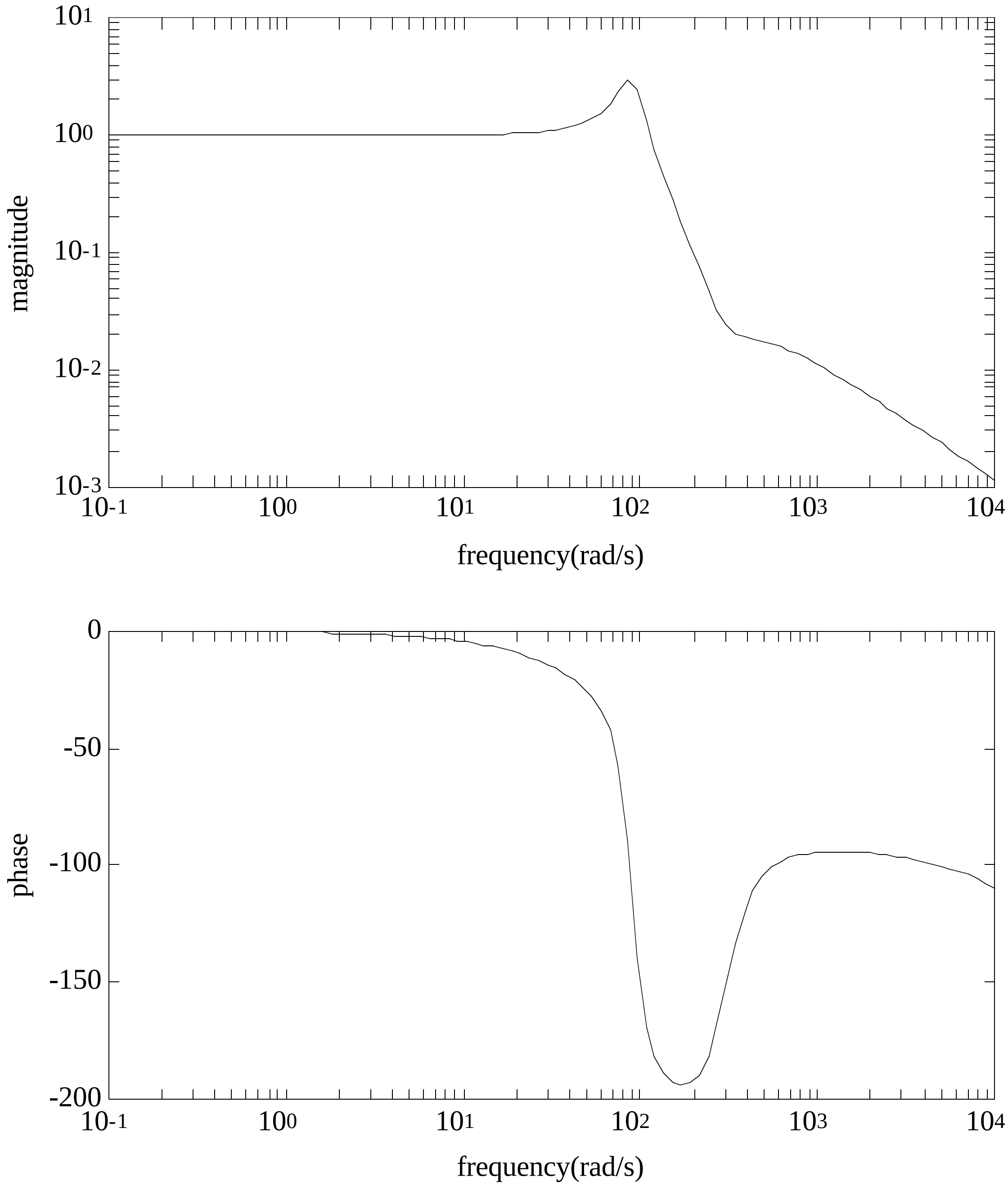


Figure 8: The resonance peak occurs near the natural frequency of the environment. The gain margin is 1.2 at $\omega = 118$rad/s, which corresponds to the root locus crossing to the right half plane in Figure 7.

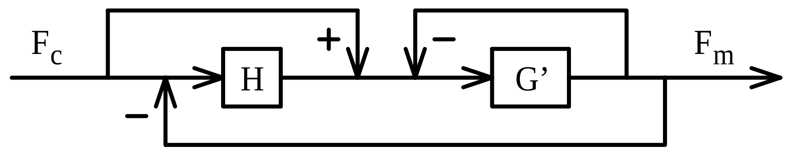


Figure 9: Block diagram a force-based explicit force controller with proportional gain and unity feedforward. The plant G has been replaced by G' and the explicit feedback path of the environmental reaction force.

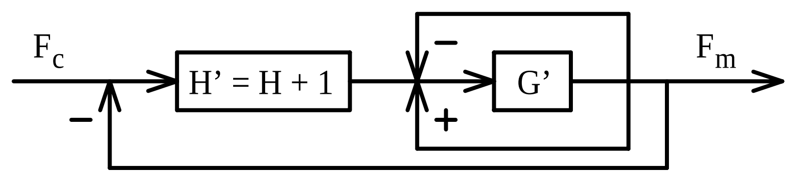


Figure 10: Block diagram a force-based explicit force controller with proportional gain and extra feedback for reaction force compensation. The plant G has been replaced by G' and the explicit feedback path of the environmental reaction force.

a resonance peak from the environment dynamics at approximately 100 rad/s. After this peak there is a 40 dB/decade drop-off which gives a minimum phase margin of $\sim 15°$ at $K_{fp} \approx 1$.

The addition of a lowpass filter in the feedback loop can improve the response by introducing a dominant pole on the real axis [1, 16]. However, this pole placement and the behavior of the system closely match that provided by integral control. A discussion including integral force control is outside the scope of this paper [16]. Therefore, lowpass filtering will not be considered further.

It will prove useful later (in the discussion of impedance control) to now discuss the feedforward term in more detail. It is usually desirable that the feedforward term be unity so that the environmental reaction force will be canceled during steady state. Figure 9 shows the block diagram of the system. The plant G' differs from G in that the reaction force has be extracted and shown explicitly in the block diagram. The transfer function of this system is:

$$\frac{F_m}{F_c} = \frac{(H+1)G'}{1+(H+1)G'} \tag{3}$$

$$= \frac{H'G'}{1+H'G'} \tag{4}$$

where $H' = H + 1$. It is seen directly that an equivalent block diagram of the system may be constructed as in Figure 10. Viewed in this way, the reaction force is negated explicitly, and the proportional gain may have values $H' = K'_{fp} \geq 0$ or $H = K_{fp} \geq -1$. Thus, the proportional gain of the original controller may be as small as negative one.

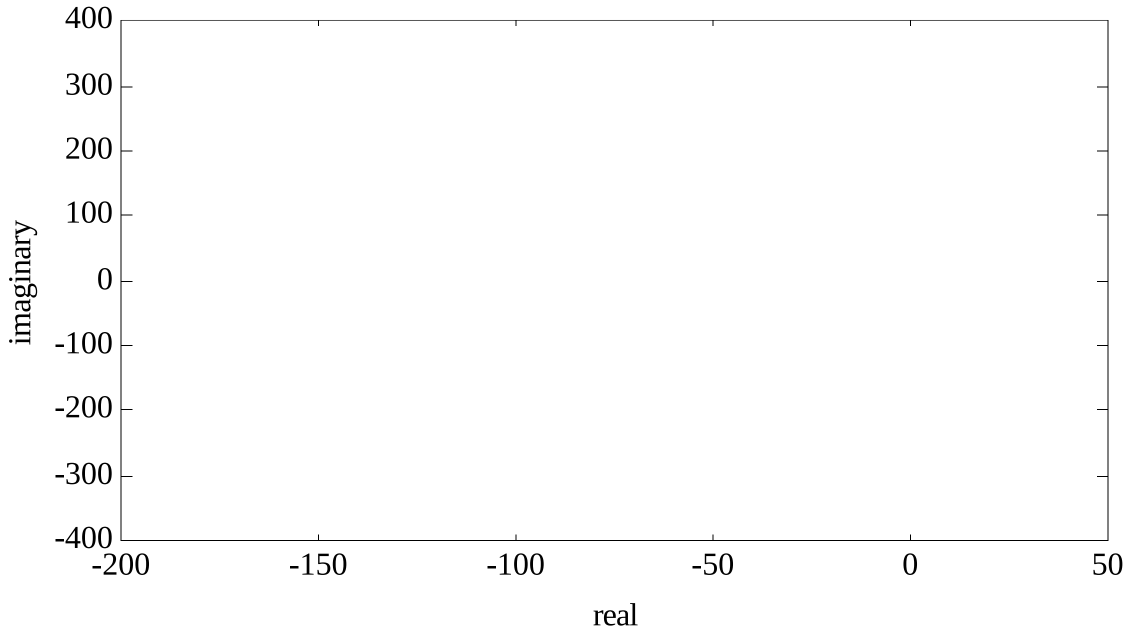


Figure 11: Root locus for the fourth order model for $-1 \leq K_{fp} < \infty$ or $0 \leq K'_{fp} < \infty$.

Figure 11 shows the proportional gain force control root locus for gains as low as negative one. The use of negative gains like this has appeared in the literature previously [5, 7]. However, this result is usually presented within the framework of impedance control. As will be seen in the following sections, the impedance controllers for which this result was obtained actually contain proportional gain explicit force control, which mandates the result.

3 Review of Impedance Control

Impedance control is a strategy that controls the dynamic relation between the manipulator and the environment. The force exerted on the environment by the manipulator is dependent on its position and its impedance. Usually this relation is expressed in Cartesian space as:

$$f = \mathcal{Z}(x). \tag{5}$$

where f, x, and $\mathcal{Z}$, are force, position, and impedance. The impedance consist of two components: that which is physically intrinsic to the manipulator, and that which is given to the manipulator by active control. It is the goal of impedance control to mask the intrinsic properties of the arm and replace them with the target impedance.

The impedance relation can have any functional form. It has been shown that general impedances are useful for obstacle avoidance [6, 10, 17]. However, it will be made clear in this section that sensor based, feedback controlled interaction with the environment requires the impedance to be linear and of second order at most. This is for two reasons. First, the dynamics of a second order system are well understood and familiar. Second, for higher order systems it is difficult to obtain measurements corresponding to the higher order state variables. These measurements are required for closed loop control.

To implement impedance control, model based control can be used. This type of scheme relies on the inverse of the Jacobian. A second type of controller which uses the transpose of the Jacobian is sometimes employed. Both forms of impedance control will be shown to contain proportional gain explicit force control (with feedforward). Also, when in contact with an environment of any appreciable stiffness, the position feedback is essentially constant and impedance control reduces directly to proportional gain force control. The role of proportional gain force control in impedance control, and their equivalence when in contact with stiff environments, has not been recognized or demonstrated previously.

The next sections are organized as follows. First, the order of the desired impedance will be discussed and the implications for implementation will be shown. Second, model based impedance control will be reviewed. The reduced form of impedance control without dynamics compensation will also be presented. Third, it will be shown how each of these schemes contain an internal proportional gain force control loop. This control loop becomes the only active one if the position feedback is constant, as in the case of contact with a stiff environment.

3.1 Zeroth, First, and Second Order Impedance

A linear impedance relation may be represented in the Laplace domain as:

$$F = Z(s)X. \tag{6}$$

The order of the polynomial $Z(s)$ is the order of the impedance.

The simplest form of an impedance controller has a zeroth order impedance. In this case Z is a constant and

$$F = KX. \tag{7}$$

The impedance parameter K is the desired stiffness of the manipulator. When a manipulator has no intrinsic stiffness, K dictates the apparent stiffness of the arm. This is accomplished with active control that uses position feedback. The value of the active stiffness is the position feedback gain.

A more typical form of an impedance controller is a first order impedance. In this case:

$$F = (Cs + K)X. \tag{8}$$

The added parameter C is the desired damping of the manipulator. It is equal to the sum of the active and natural damping. The active damping is accomplished by velocity feedback in a position controlled system. The value of the active damping is the velocity feedback gain. Since the active damping can be modified, C can take on any value which maintains stability. In fact, negative active damping can be used to eliminate the appearance of any damping in the arm. This is rarely desirable, since damping has a stablizing effect.

The last form of impedance control that shall be considered here is a second order type. The second order impedance controller has the form:

$$F = \left(Ms^2 + Cs + K\right) X. \tag{9}$$

The parameter M is the desired inertia of the manipulator. While the arm has an intrinsic inertia due to its mass, this can be modified by active feedback. It follows from the previous two cases that acceleration feedback can be used for this purpose. In this case, the value of active inertia is the acceleration feedback gain. Its value can be used to adjust M. Few researchers have proposed such acceleration feedback schemes for impedance control [4]. This is because an acceleration measurement typically requires a second derivative, which will be extremely noisy. Alternatively, the force may be measured and the acceleration commanded. This is typically the method employed, as will be shown.

3.2 Model Based Control

Model based control involves the use of a dynamic model of the manipulator to determine the actuation torques [2]. Model based impedance control may be summarized by the following equations [6, 16]:

$$\begin{aligned} \tau_A &= Du + h + g + J^T f_m && (10) \\ u &= J^{-1}\left(\ddot{x}_u - \dot{J}\dot{\theta}_m\right) && (11) \\ \ddot{x}_u &= M^{-1}\left[(C\Delta\dot{x} + K\Delta x) - f_m\right] && (12) \\ \Delta x &= x_c - \mathcal{F}(\theta_m) && (13) \\ \Delta\dot{x} &= \dot{x}_c - J\dot{\theta}_m. && (14) \end{aligned}$$

Equation (10) describes the dynamics of the manipulator with inertia matrix D, Coriolis and centripetal force vector h, and gravitational force vector g. Equation (11) describes the control signal in terms of the desired Cartesian acceleration. Equation (12) specifies the desired second order impedance control relationship. Equations (13) and (14) determine the Cartesian position and velocity errors through the forward kinematics, $\mathcal{F}$, and the manipulator Jacobian, J. The subscripts c and m indicate commanded and measured quantities.

Without force feedback this control scheme is equivalent to position control schemes such as Resolved Acceleration Control [13] and Operational Space Control [9]. These are only first order impedance control schemes since they just modify the stiffness and damping of the arm. Including force feedback information in the controller yields second order impedance control [6]:

$$\tau = DJ^{-1}M^{-1}\left(C\Delta\dot{x} + K\Delta x - f_m\right) - J^{-1}\dot{J}\dot{\theta} + h + g + J^T f_m. \qquad (15)$$

or

$$\tau = J^T\Lambda M^{-1}\left(C\Delta\dot{x} + K\Delta x - f_m\right) - J^T\Lambda\dot{J}\dot{\theta} + h + g + J^T f_m. \qquad (16)$$

where

$$D(\theta) = J^T\Lambda(x)J \qquad (17)$$

and the matrix, Λ, is the Cartesian space representation of the inertia matrix. The first form of the controller is necessary if the inverse dynamics calculations expressed in Equation (10) are used. In this case, the inverse of the Jacobian and the arm inertia must be calculated. The second form is useful as a steady state approximation in which the manipulator inertia does not change or is not known [8]. ($\dot{J}$ as well as h may equal zero also.) In this second case, the inverse of the Jacobian need not be calculated; only its transpose is necessary. The arm inertia need not be calculated either, since only its product with the impedance mass parameter is needed, as will be explained shortly.

Note that the force feedback is used in two places. First, this feedback is used in the physical model of the arm dynamics, Equation (10). This is equivalent to introducing end effector forces into the Newton-Euler dynamics calculations. Second, the feedback is used in the impedance relation, Equation (12). While Equation (10) effectively linearizes the arm, Equation (12) modifies the impedance control signal to compensate for the experienced force.

It can now be seen that it is the force feedback in the control signal which modifies the apparent inertia of the arm [7]. Equation (16) best shows this effect. In this controller, as with the previous ones, the premultiplication of K and C by ΛM^{-1} changes nothing; $\Delta\dot{x}$ and Δx are still multiplied by a gain. However, things are made different by the force feedback signal f_m. It is multiplied by the term ΛM^{-1}, which is a mass ratio that reduces or increases the amount of actuator torque applied. For simplicity sake, it will be assumed that the impedance parameters are diagonal in the Cartesian space defined by the eigenvectors of Λ. In this case, ΛM^{-1} (or its diagonalized counterpart) can be thought of as a matrix of mass ratios along the diagonal. Since Λ is due to the physical inertia of the arm, it is the impedance parameter M which determines each ratio. For $M \to 0$ the ratio becomes very large; for a small measured force, a large accelerating torque is applied to the arm. Thus, the apparent inertia of the arm is reduced. (It is important to remember that the external force does not contribute to the acceleration because it has been effectively negated by the $J^T f_m$ term.) Similarly, for $M \to \infty$ the ratio becomes very small; for a large measured force, a small accelerating torque is applied to the arm. Thus, the apparent inertia of the arm is increased. In this way, second order impedance control not only changes the stiffness and damping properties of the arm, but its inertia as well.

3.3 Explicit Force Control within Impedance Control

The two second order impedance controllers reviewed above can be shown to contain explicit force control. This aspect of impedance control has not previously been recognized. While some correspondence between impedance control and explicit force control has been recognized, the relation was not specifically or clearly stated [7]. A general argument supporting this new interpretation was presented in the introduction. Now, it will be shown explicitly for the impedance controllers described previously in this paper. It has been shown elsewhere how this framework includes both Stiffness Control, and Accommodation Control [16].

Consider the second order impedance controller represented by Equation (16). This can be rewritten in the form:

$$\begin{aligned} \tau &= J^T \left[K'_{fp}(f_c - f_m) + f_m - K'_v \dot{x}_m \right] + g && (18) \\ f_c &= K(x_c - x_m) + C\dot{x}_c && (19) \\ K'_{fp} &= \Lambda M^{-1} && (20) \\ K'_v &= \Lambda M^{-1} C && (21) \end{aligned}$$

$$(22)$$

This formulation is very similar to the proportional gain explicit force controller discussed previously and corresponds to the block diagram shown in Figure 10 with $K'_{fp} = H'$. Since velocity feedback was also used in the proportional gain explicit force controller, the only major difference here is the use of the feedback position in the calculation of the commanded force. The commanded velocity can be assumed to be zero ($\dot{x}_c = 0$), which is usually the case.

In the above formulation, the impedance parameters K and C determine the commanded force used in an explicit force controller with a proportional gain of $K'_{fp} = \Lambda M^{-1}$. The term $J^T f_m$ can be seen to negate the reaction force that is experienced by the arm. As expressed in the transfer function of Equation (4) the stability of this controller is guaranteed for $K'_{fp} \geq 0$. This is equivalent to the condition:

$$\Lambda M^{-1} \geq 0 \tag{23}$$

(Again, it is assumed that the matrices K'_{fp} and ΛM^{-1} are diagonal. Therefore the inequality is considered to refer to each of the elements individually.) This loosely implies that the open loop pole location of the root locus corresponds to the impedance parameter $M \to \infty$ and the zeros indicate a value of $M \to 0$. Of course, this statement is only true if the principal axes of Λ and M are parallel.

It is also important to note that force control is often used when the manipulator is in contact with a stiff environment; $\dot{x}_m = 0$, and x_m is an arbitrary constant which can be set to zero. Again, it is the usual case that the commanded velocity is zero ($x_c = 0$). Thus, the commanded force reduces to:

$$f_c = K x_c \tag{24}$$

which has no dependence on position feedback. In this case, K acts merely as a scaling factor to the commanded position. This scaled commanded position can be directly replaced by commanded force. With force as the commanded quantity Equation (19) can be eliminated, and the controller becomes simply Equation (18). Since the the gains K'_{fp} and K'_v are independently adjustable, this controller has an identical structure to the explicit force controller in Equation (1). Therefore, is apparent that second order impedance control against a stiff environment is equivalent to explicit force control with proportional gain and feedforward. In the following section it will be seen that experimental results confirm this conclusion.

4 Experimental Results

This section presents the experimental results of implementations of proportional gain force control with feedforward, and second order impedance control, with and without dynamics compensation. It will be seen that in each case, the response and stability of the system are essentially the same.

All experiments were conducted using the CMU DD Arm II and implemented under the Chimera II real time operating system [15]. The experiments presented here were conducted by pushing on a common environment: a cardboard box with an aluminum plate resting on top. The parameters of a second order model of this environmental system were: stiffness $k \approx 10^4$ N/m, damping $c \approx 17$ N $\cdot$ s/m, and mass $m \approx 0.1$ kg. The control rate was 300 Hz, except in the case of dynamics compensation, where it was 250Hz. In all experiments the velocity gain was $K_v = 10$. All graphs of data show the reference values as a dashed line and the measured values as a solid line.

4.1 Proportional Gain with Feedforward Control

The first controller to be discussed is proportional gain force control with the reference force fedforward. Figures 12 (a) through (h) show the response of this controller to the commanded force trajectory. There are several things to note about the response profiles to variations in the proportional gain. First, as predicted by the model, the system exhibits the characteristics of a Type 0 system: finite steady state error for a step input and unbounded error for a ramp input. Second, for an increase in position gain, the steady state error reduces, but at the cost of increasingly larger overshoot. As correctly predicted by the root locus of the system model in Figure 7, this control scheme causes instability at $K_{fp} \approx 1$. Also, the fact that the environmental poles are always off the real axis can be seen in the steady state oscillations that occur at the system's natural frequency (~ 15 Hz), particularly after the step input. Finally, it can be seen that negative proportional gains are increasingly more stable, but the response of the system approaches zero as $K_{fp} \rightarrow -1$.

4.2 Impedance Control

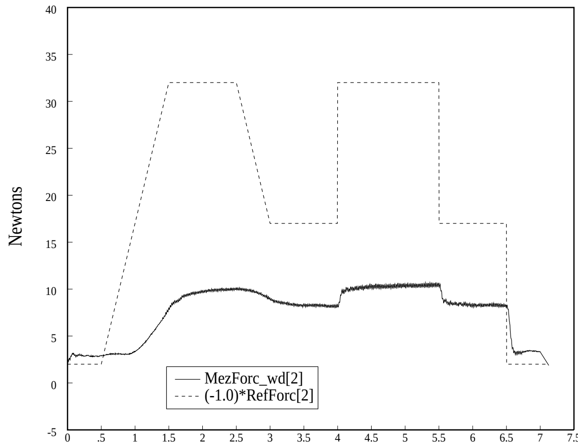
This section presents the results of implementing the second order impedance control schemes presented earlier. The position reference trajectories are chosen such that given the stiffness of the controller, the trajectory should provide the same force profile as commanded for the proportional gain explicit force controller, thus allowing a direct comparison with that controller. As will become apparent, both forms of impedance control (with and without dynamics compensation) respond the same as proportional gain explicit force control with feedforward.

4.2.1 Impedance Control Without Dynamics Compensation

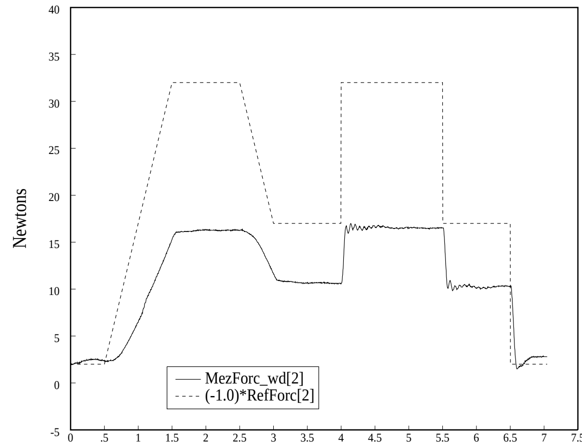
When in contact with a stiff environment, the manipulator will not move very much or very quickly in the direction normal to the environment. As was shown previously, this enables a steady state approximation that eliminates the need to calculate the inverse dynamics and the inverse Jacobian. The control law has the form of Equation (18). For these experiments $K = 150$ and $C = 10$. As discussed in Section 3.3 the mass ratio, ΛM^{-1}, is equivalent to the proportional force gain plus one:

$$\Lambda M^{-1} = K_{fp} + 1. \tag{25}$$

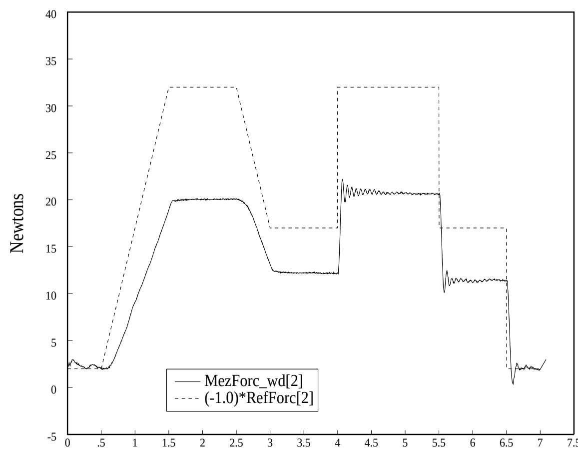
Thus, for this controller the mass ratio is chosen to be diagonal in the desired frame and its components (λ/m) set to correspond to the proportional force gains chosen previously. This allows a direct comparison of impedance and force control schemes.



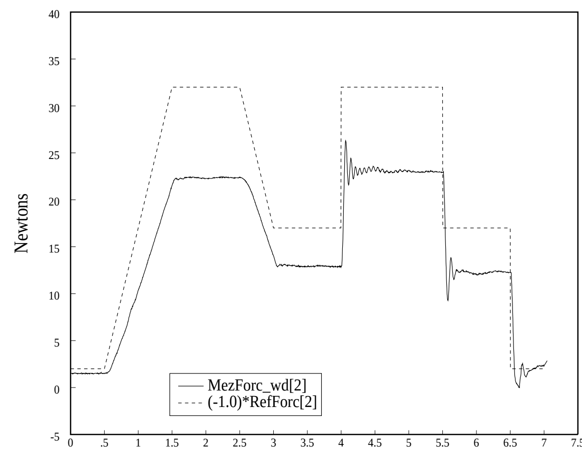
(a) $K_{fp} = -0.75$



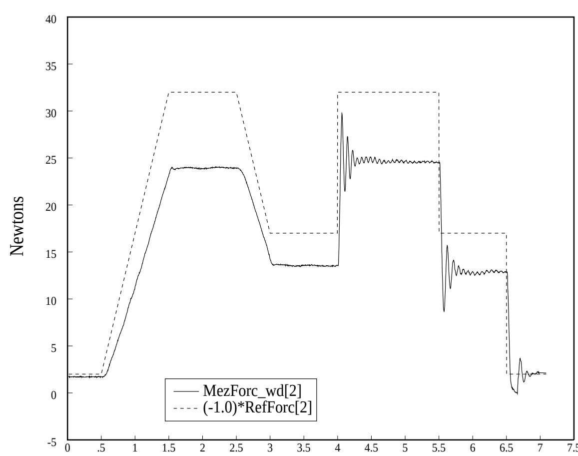
(b) $K_{fp} = -0.5$



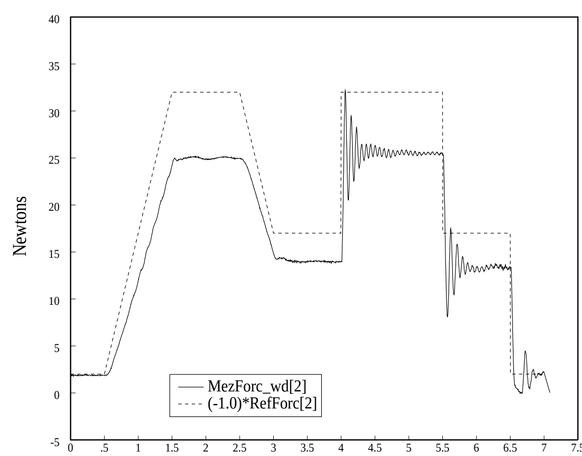
(c) $K_{fp} = -0.25$



(d) $K_{fp} = 0$

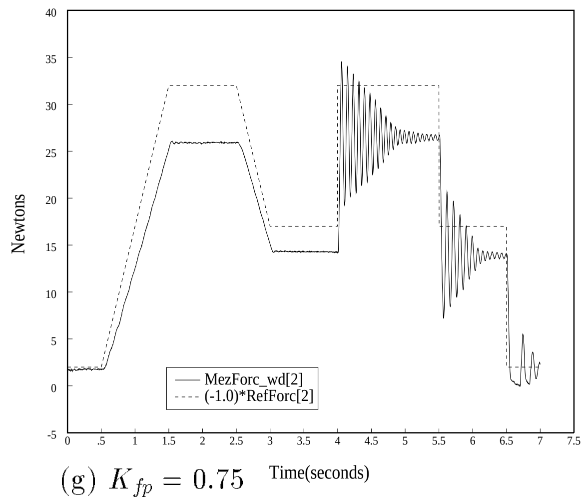


(e) $K_{fp} = 0.25$

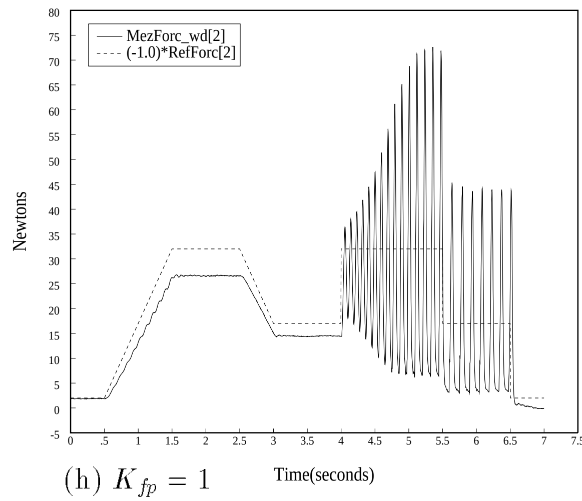


(f) $K_{fp} = 0.5$

Figure 12: Experimental data of proportional gain explicit force control with feedforward. The proportional gain varies from -0.75 to 1.



(g) $K_{fp} = 0.75$



(h) $K_{fp} = 1$

Figure 12: (continued) Experimental data of proportional gain explicit force control with feedforward. The proportional gain varies from -0.75 to 1.

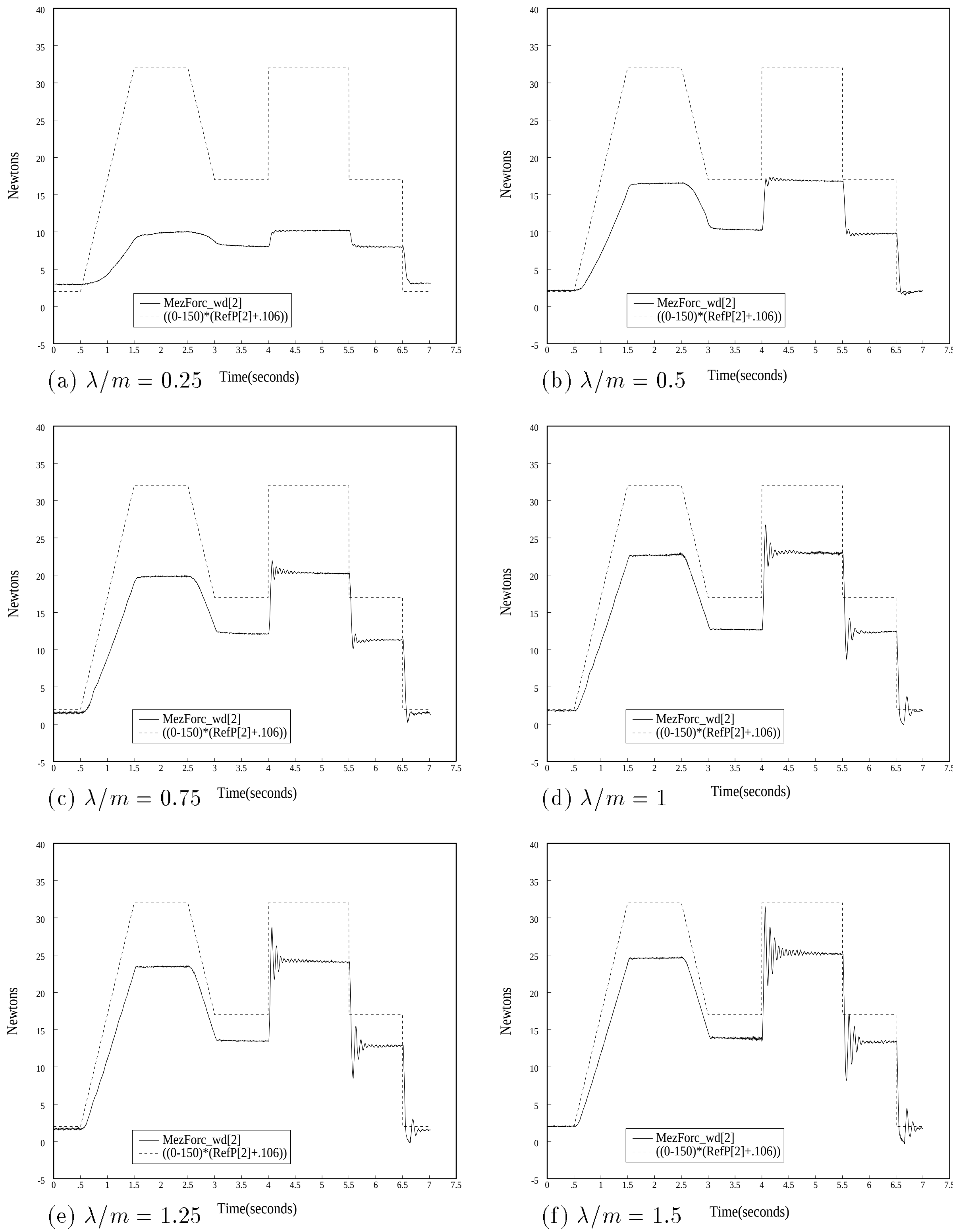
Figures 13 (a) through (h) show the response of this impedance controller, as well as the commanded position trajectory multiplied by the active stiffness in that direction. As is readily apparent, the response of this controller is essentially equivalent to that of the proportional gain controller shown in Figures 12 (a) through (h). This confirms the previous theoretical assertion that second order impedance control against a stiff environment is equivalent to explicit force control with proportional gain and feedforward compensation.

4.2.2 Impedance Control With Dynamics Compensation

Second order impedance control can also be implemented with dynamics compensation as shown in Equation (15). As can be seen in Equation (16), the mass ratio, ΛM^{-1} can be thought of a proportional force gain. However, it can be seen from Equations (11) and (12) that only M can be specified in this scheme.

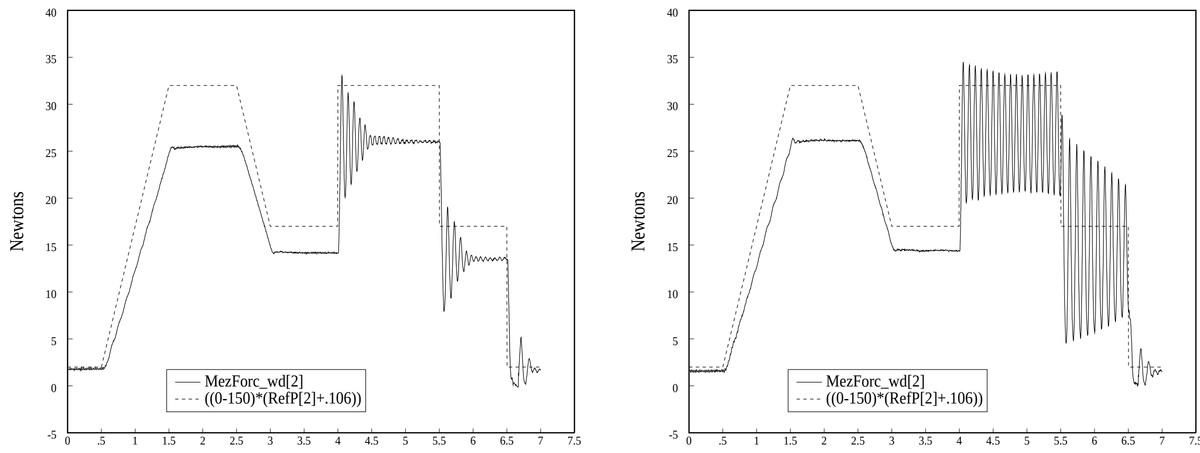
Usually M is chosen to be diagonal in the task frame along with K and C. When operating in free space ($f_m = 0$) a diagonal M acts as a simple scaling factor for K and C, thereby preventing coupled motion. If M were nondiagonal, its product with diagonal K and C would be nondiagonal, and coupled motion would result. Further, K and C are usually chosen to be diagonal in some task frame which is aligned with the environment to be contacted. In this way, the manipulator may be made stiff tangential to a surface, but soft normal to it. (The velocity gains are usually chosen for critical damping.)

However, when in contact with the environment ($f_m \neq 0$), the ratio of the inertias, ΛM^{-1}, acts as a proportional force gain which is not diagonal in general, because Λ is not diagonal in general. Therefore, it is necessary to determine the effective value of the mass ratio (force gain). This requires finding the dominant element of Λ for the direction in which the environment is contacted.



(a) $\lambda/m = 0.25$
(b) $\lambda/m = 0.5$
(c) $\lambda/m = 0.75$
(d) $\lambda/m = 1$
(e) $\lambda/m = 1.25$
(f) $\lambda/m = 1.5$

Figure 13: Experimental data of impedance control without dynamics compensation. The mass ratio 'gain' varies from 0.25 to 2.0.



(g) $\lambda/m = 1.75$ (h) $\lambda/m = 2$

Figure 13: (continued) Experimental data of impedance control without dynamics compensation. The mass ratio 'gain' varies from 0.25 to 2.0.

Finding the dominant component of the inertia matrix is equivalent to finding the effective mass in the direction of concern. Since it is the force which is being controlled, this can only be done by determining the resultant acceleration from an applied force:

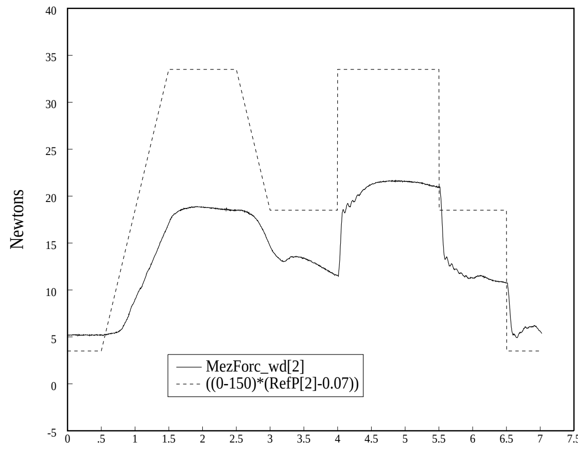
$$\ddot{x} = \Lambda^{-1} f \tag{26}$$

The force may be set to be the unit vector in the direction of the surface. For the experiments performed, the z direction was chosen. The actual values of Equation (26) were:

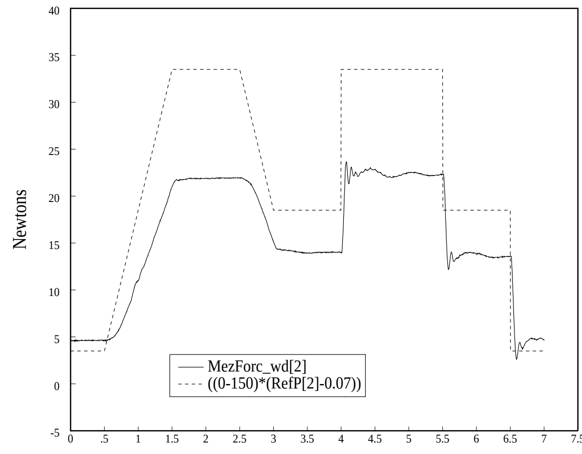
$$\ddot{x} = \begin{bmatrix} 0.070 & 0 & -0.053 & 0 & 0 & 0 \\ 0 & 1.671 & 0 & -9.723 & -0.049 & 0 \\ -0.053 & 0 & 0.199 & 0 & 0 & 0 \\ 0 & -9.723 & 0 & 59.9 & -1.272 & 0 \\ 0 & -0.049 & 0 & -1.272 & 2.758 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3226 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{27}$$

It is apparent that for forces applied in the z direction to the arm in this configuration, the dominant acceleration is $\ddot{x}_z \approx 0.2\mathrm{m/s}^2$. Thus, the apparent inverse scalar mass is $\lambda_z^{-1} \approx 0.2\ \mathrm{kg}^{-1}$. This implies that the best scalar approximation of the mass in the z direction is $\lambda_z \approx 5$ kg. This value may then be thought of as a scaling factor applied to the variable gain value M_{33} in Equation (15).

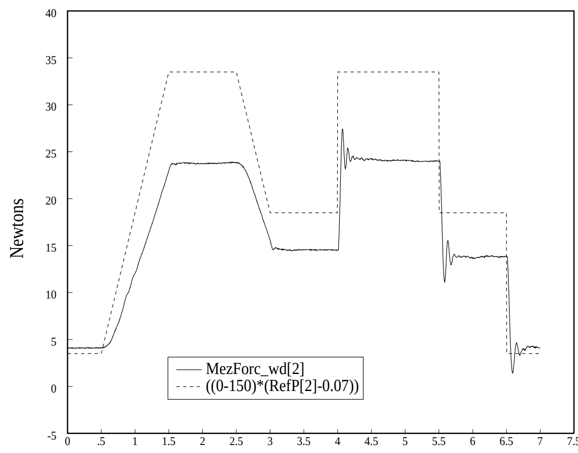
Figures 14 show the response of impedance control with dynamics compensation for $0.1 \leq M_{33}^{-1} \leq 0.45$. The other gains were $K = 150$ and $C = 10$. Using the approximation of $\lambda_z \approx 5$ kg, these gains can be thought of as $0.5 \leq \lambda_z M_{33}^{-1} \leq 2.25$. Thus, a direct comparison can be made between the system response shown in Figures 14 and that shown



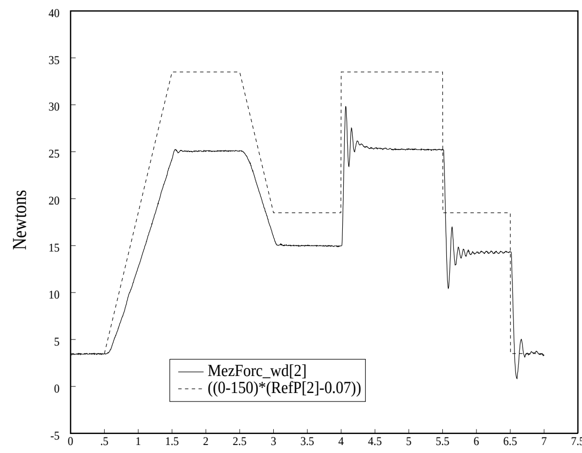
(a) $M^{-1} = 0.1$



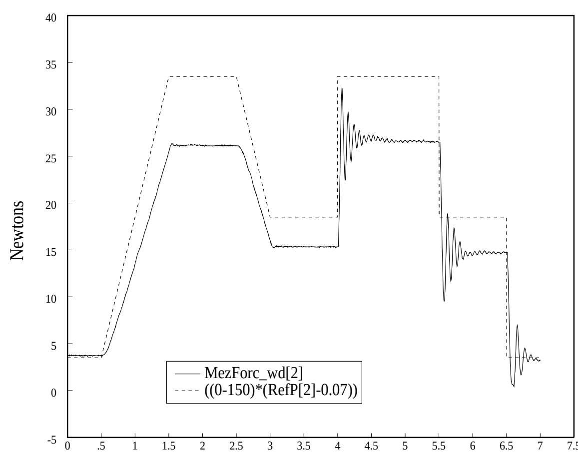
(b) $M^{-1} = 0.15$



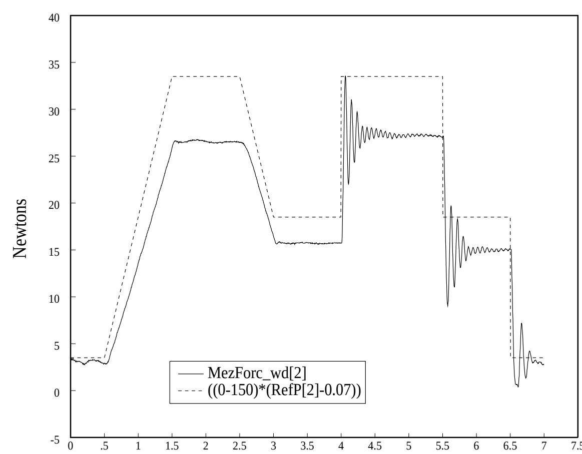
(c) $M^{-1} = 0.2$



(d) $M^{-1} = 0.25$

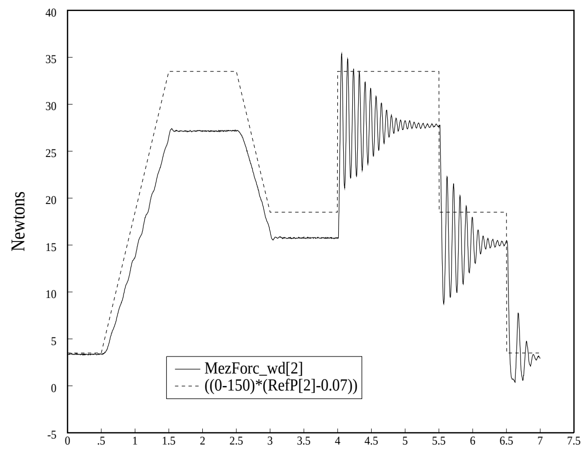


(e) $M^{-1} = 0.3$

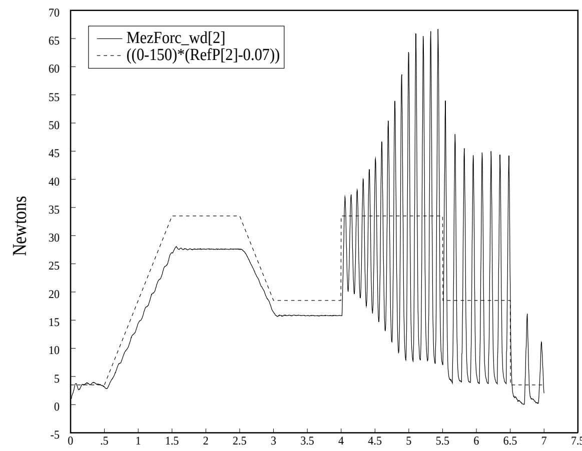


(f) $M^{-1} = 0.35$

Figure 14: Experimental data of impedance control with dynamics compensation. The commanded inverse mass varies from 0.1 to 0.45. This is approximately the same as having a force gain vary from 0.5 to 2.25.



(g) $M^{-1} = 0.4$



(h) $M^{-1} = 0.45$

Figure 14: (continued) Experimental data of impedance control with dynamics compensation. The commanded inverse mass varies from 0.1 to 0.45. This is approximately the same as having a force gain vary from 0.5 to 2.25.

in Figures 13 and Figures 12. The responses are essentially the same. This comparison supports the analysis above. It also, further supports the conclusion that second order impedance control, with or without dynamics compensation, is essentially equivalent to explicit force control with proportional gain and feedforward compensation.

5 Conclusions

It is apparent from the results presented here that second order impedance control and proportional gain explicit force control with feedforward are essentially the same thing. This leads us to the question the value of impedance control as a unified controller for motion through, and constrained interaction with, the environment. Our conclusion that impedance control is not the best solution for these modes of operation is illustrated by the following discussion.

First, it has been shown the proportional gain force control is not the best force controller; integral gain control provides much better tracking [16]. Therefore, the behavior of the impedance controller while in contact with the environment is not optimal, and not always stable.

Second, impedance control is more cumbersome to use since it requires position reference instead of force reference. Some researchers see this as a strength since there is no need to switch inputs between the modes of free space motion and constrained force application. However, this implies there is knowledge of the position commands necessary for an operation that intrinsically requires force commands.

Third, while not in contact with the environment, impedance control continues to incorporate force feedback information into the control law. Phenomena such as sensor noise or inertial loading can cause nonzero force readings and inhibit the performance of the position control.

Fourth, impedance control gains that are stable during unconstrained and constrained actuation cause oscillation or instability during the transition phase of impact [16]. Adaptively modifying the gains may provide a fix, but detracts from the notion that impedance control can work in all manipulation situations. Further, if switching is to be employed it seems attractive to switch controllers as well as gains, and get the best performance possible from the system. Stable response through the impact transient has been demonstrated by switching control schemes [19, 11, 12].

Therefore, the results of this work indicate two major points. First, second order impedance control must be recognized as essentially equivalent to proportional gain explicit force control with feedforward. And second, if impedance control is to be used it has inherent limitations that make it something less than the best controller for any given manipulation mode.

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